Malaysian Journal of Mathematical Sciences 10(S) August: 267–277 (2016)
Special Issue: The 7^{th} International Conference on Research and Education in Mathematics (ICREM7)

MALAYSIAN JOURNAL OF MATHEMATICAL SCIENCES

Journal homepage: http://einspem.upm.edu.my/journal

Optimal Pursuit Time in Differential Game for an Infinite System of Differential Equations

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ABSTRACT

We consider a differential game of one pursuer and one evader. The game is described by an infinite system of first order differential equations. Control functions of the players are subject to coordinate-wise integral constraints. Game is said to be completed if each component of state vector equal to zero at some unspecified time. The pursuer tries to complete the game and the evader pursues the opposite goal. A formula for optimal pursuit time is found and optimal strategies of players are constructed.

Keywords: Infinite system of differential equations, pursuer, evader, strategy, optimal pursuit time.

1. Introduction

There are many works devoted to differential games with integral constraints in finite dimensional spaces such as Krasovskii (1968), Mezentsev (1971), Ushakov (1972), Azimov (1974), Subbotin and Ushakov (1975), Okhezin (1977), Gusiatnikov et al. (1979), Satimov et al. (1983), Solomatin (1984), Konovalov (1987), Ukhobotov (1987), Lokshin (1990), Ibragimov (1998), Guseinov et al. (1999), Azamov and Samatov (2000), Ibragimov (2005), Chikrii and Belousov (2009), Belousov (2011), Ibragimov and Satimov (2012), Chikrii and Belousov (2013) and Kuchkarov (2013).

However, there a few works devoted to differential game problems described by infinite system of differential equations. For example, the following papers are devoted to such game problems Li (1986), Tukhtasinov (1995), Ibragimov (2002), Satimov and Tukhtasinov (2005), Satimov and Tukhtasinov (2005), Tukhtasinov (2005), Satimov and Tukhtasinov (2006), Satimov and Tukhtasinov (2007), Ibragimov and Hussin (2010), Azamov and Ruziboyev (2013), Ibragimov (2013), Ibragimov et al. (2014) and Ibragimov et al. (2015), Salimi et al. (2015).

In the paper Ibragimov et al. (2015), a pursuit game problem is studied for an infinite system of differential equations, where control functions of players are subjected to coordinate-wise integral constraints. In the present paper, we prove that guaranteed pursuit time found in Ibragimov et al. (2015) is optimal pursuit time as well.

2. Statement of problem

Consider differential game described by the following first order infinite system of differential equations

$$\dot{z}_i + \lambda_i z_i = u_i - v_i,\ z_i(0) = z_{i0}\ i = 1, 2, ..., \tag{1}$$

where $z_i, u_i, v_i \in \mathbb{R}^{n_i}$, n_i is a positive integer, λ_i are given positive numbers, $u = (u_1, u_2, ...)$ and $v = (v_1, v_2, ...)$ are control parameters of the pursuer and evader respectively. Let $T > 0$ be an arbitrary number.

Definition 2.1. *A function $u(t) = (u_1(t), u_2(t), ...)$, $0 \le t \le T$, with measurable coordinates $u_i(t) \in \mathbb{R}^{n_i}$, is called admissible control of the i-th pursuer if it satisfies the following integral constraint*

$$\int_0^T |u_i(s)|^2 ds \le \rho_i^2, \tag{2}$$

where ρ_i is a given positive number.

Definition 2.2. *A function $v(t) = (v_1(t), v_2(t), ...)$, $0 \leq t \leq T$, with measurable coordinates $v_i(t) \in \mathbb{R}^{n_i}$, is called admissible control of the evader if it satisfies the following integral constraint*

$$\int_0^T |v_i(s)|^2 ds \leq \sigma_i^2, \tag{3}$$

where σ_i is a given positive number.

Pursuit starts from the initial positions $z_i(0) = z_{i0}$, $i = 1, 2, ...$ at time $t = 0$ where $z_{i0} \in \mathbb{R}^{n_i}$, $i = 1, 2, ...$

Next, we define solution of the system (1). If we replace the parameters u_i and v_i in the equation (1) by some admissible controls $u_i(t)$ and $v_i(t)$, $0 \leq t \leq T$, then it follows from the theory of differential equations that the initial value problem (1) has a unique solution on the time interval $[0, T]$. The solution

$$z(t) = (z_1(t), z_2(t), ...), 0 \leq t \leq T,$$

of infinite system of differential equations (1) is considered in the space of functions $f(t) = (f_1(t), f_2(t), ...)$ with absolutely continuous coordinates $f_i(t)$ defined on the interval $0 \leq t \leq T$.

Define strategy of the pursuer.

Definition 2.3. *A function of the form*

$$U(t, v) = \psi(t) + v = (\psi_1(t) + v_1, \psi_2(t) + v_2, ...), \ 0 \leq t \leq T,$$

where $U_i(t, v_i) = \psi_i(t) + v_i \in \mathbb{R}^{n_i}$, $i = 1, 2, ...,$ is called strategy of the pursuer if for any admissible control of the evader $v = v(t)$, $0 \leq t \leq T$, the following inequalities

$$\int_0^T |U_i(t, v_i(t))|^2 dt \leq \rho_i^2, i = 1, 2, ...,$$

hold, where $\psi(t) = (\psi_1(t), \psi_2(t), ...)$, $0 \leq t \leq T$, is a function with measurable coordinates $\psi_i(t) \in \mathbb{R}^{n_i}$.

Definition 2.4. *We say that pursuit can be completed for the time $\theta > 0$ in the differential game (1)–(3) from the initial position $z_0 = (z_{10}, z_{20}, ...)$ if there exists a strategy of the pursuer $U(t, v)$ such that for any admissible control of the evader $v_i(t)$, $0 \leq t \leq \theta$, the solution $z(t)$, $0 \leq t \leq \theta$, of the initial value problem*

$$\begin{aligned} &\dot{z}_i + \lambda_i z_i = U_i(t, v(t)) - v_i(t), \ 0 \leq t \leq \theta, \\ &z_i(0) = z_{i0}, \ i = 1, 2, ..., \end{aligned}$$

equals zero at some time τ, $0 \leq \tau \leq \theta$, i.e. $z(\tau) = 0$.

In the sequel, such a time θ is called guaranteed pursuit time.

To define strategy of the evader, we need the variables $p(t) = (p_1(t), p_2(t), ...)$, and $q(t) = (q_1(t), q_2(t), ...)$, which are defined as the solutions of the following equations

$$\dot{p}_i = -u_i^2(t), p_i(0) = \rho_i^2, \dot{q}_i = -v_i^2(t),\ q_i(0) = \sigma_i^2, i = 1, 2, ...$$

Clearly,

$$p_i(t) = \rho_i^2 - \int_0^t u_i^2(s)ds,$$

and

$$q_i(t) = \sigma_i^2 - \int_0^t v_i^2(s)ds.$$

The quantity $\int_0^t u_i^2(s)ds$ expresses the amount of energy spent by the pursuer in the i-th component of control u_i. The quantity $p_i(t)$ is the amount of energy remained for the i-th component of control u_i which the pursuer can use starting from time t. The quantities $\int_0^t v_i^2(s)ds$ and $q_i(t)$ also have similar meanings for the evader.

Definition 2.5. *A function $V(t) = (V_1(t), V_2(t), ...),\quad t \geq 0$, with the coordinates of the form*

$$V_i(t) = \begin{cases} V_{i0}(t) & 0 \leq t \leq \tau_i, \quad q_{i0}(\tau_i) \geq 0, \\ 0 & \tau_i < t \leq \tau_i + \varepsilon_i, \\ u_i(t - \varepsilon_i) & t > \tau_i + \varepsilon_i, \end{cases}$$

is called strategy of evader, where $V_{i0}(t),\ t \geq 0$, is a measurable function,

$$q_{i0}(t) = \sigma_i^2 - \int_0^t V_{i0}^2(s)ds,$$

$t = \tau_i$ is the first time for which $p_i(t) = q_{i0}(t)$, $u(t) = (u_1(t), u_2(t), ...)$ is any admissible control of pursuer, ε_i is a positive number.

Definition 2.6. *A guaranteed pursuit time θ is called optimal pursuit time if there exists a strategy of the evader V such that for any admissible control of the pursuer $z(t) \neq 0$ for all $t \in [0, \theta)$.*

Problem 1. *Find optimal pursuit time in the game (1) – (3), and construct the strategy for pursuer that enables to complete the game for this time.*

3. Main result

The following theorem presents a formula for optimal pursuit time in the game (1) – (3).

Theorem 3.1. *Let $\rho_i > \sigma_i$, $i = 1, 2, ...,$ and $\sup\limits_{i \in N} \frac{|z_{i0}|}{\rho_i - \sigma_i} < \infty$. Then*

$$T' = \sup T_i, \; T_i = \frac{1}{2\lambda_i} \ln \left(1 + 2\lambda_i \left(\frac{|z_{i0}|}{\rho_i - \sigma_i} \right)^2 \right) \tag{4}$$

is optimal pursuit time in the game (1)–(3).

Proof. Earlier by Ibragimov et al. (2015) was shown that T' is guaranteed pursuit time in the game (1) – (3). Therefore it is sufficient to show that on the interval $[0, T')$ evasion is possible.

3.1 Construction the strategy of the evader.

We construct it in two steps. The first part of strategy is as follows:

$$v_i(s) = -\frac{z_{i0}}{|z_{i0}|} \cdot \frac{e^{\lambda_i s}}{\varphi_i(0, T_i)} \sigma_i, \; 0 \le s \le \tau_i, \; i = 1, 2, ..., \tag{5}$$

where τ_i, $\tau_i < T_i$, is some time at which $q_i(\tau_i) = p_i(\tau_i)$,

$$\varphi_i(a, b) = \sqrt{\int_a^b e^{2\lambda_i s} ds}.$$

Note that at the time $t = 0$, $p_i(0) = \rho_i^2 > \sigma_i^2 = q_i(0)$. The evader uses (1) unless $q_i(\tau_i) = p_i(\tau_i)$ at some time τ_i. Starting from the time τ_i the evader uses the second part of the strategy. It is as follows:

$$v_i(t) = \begin{cases} 0, & \tau_i \le t \le \tau_i + \varepsilon_i, \\ u_i(t - \varepsilon_i), & t > \tau_i + \varepsilon_i, \end{cases} \quad i = 1, 2, ..., \tag{6}$$

where ε_i is a positive number.

3.2 Proof that evasion is possible.

Show that constructed strategy (5)–(6) guarantees the evasion on the time interval $[0, T)$. Let the evader use the first part of its strategy (5). Show that

$z_i(t) \neq 0$ as long as $q_i(t) \neq p_i(t)$, or more precisely $q_i(t) < p_i(t)$. Assume the contrary. Let there exist a control of the pursuer $u_i(t)$ such that $z_i(\tau_i') = 0$ at some $0 \leq \tau_i' \leq \tau_i$, where $q_i(\tau_i') \leq p_i(\tau_i')$. Then substituting (5) into the formula

$$z_i(t) = e^{-\lambda_i t}\left(z_{i0} + \int_0^t e^{-\lambda_i s}(u_i(s) - v_i(s))ds\right)$$

yields

$$z_{i0} + \int_0^{\tau_i'} e^{\lambda_i s}u_i(s)ds + \frac{z_{i0}}{|z_{i0}|} \cdot \frac{\sigma_i}{\varphi_i(0, T_i)} \cdot \int_0^{\tau_i'} e^{2\lambda_i s}ds = 0.$$

We have

$$\int_0^{\tau_i'} e^{\lambda_i s}|u_i(s)|ds \geq |z_{i0}|\left(1 + \frac{\sigma_i \cdot \varphi_i^2(0, \tau_i')}{|z_{i0}|\,\varphi_i(0, T_i)}\right). \tag{7}$$

Use the Cauchy-Schwartz inequality to estimate left hand side of (7)

$$\int_0^{\tau_i'} e^{\lambda_i s}|u_i(s)|ds \leq \varphi_i(0, \tau_i') \cdot \left(\int_0^{\tau_i} |u_i(s)|^2 ds\right)^{1/2}.$$

Then from this and (7) we obtain

$$\int_0^{\tau_i'} |u_i(s)|^2 ds \geq \frac{|z_{i0}|^2}{\varphi_i^2(0, \tau_i')}\left(1 + \frac{\sigma_i \varphi_i^2(0, \tau_i')}{|z_{i0}|\,\varphi_i(0, T_i)}\right)^2 \tag{8}$$

According to (5)

$$\int_0^{\tau_i'} |v_i(s)|^2 ds = \frac{\sigma_i^2 \varphi_i^2(0, \tau_i')}{\varphi_i^2(0, T_i)}. \tag{9}$$

Combining (8) and (9), we obtain

$$\begin{aligned}
\int_0^{\tau_i'} |u_i(s)|^2 ds - \int_0^{\tau_i'} |v_i(s)|^2 ds &= \frac{|z_{i0}|^2}{\varphi_i^2(0, \tau_i')} + 2 \cdot \frac{|z_{i0}|\,\sigma_i}{\varphi_i(0, T_i)} \\
&> \frac{|z_{i0}|^2}{\varphi_i^2(0, T_i)} + 2\sigma_i \cdot \frac{|z_{i0}|}{\varphi_i(0, T_i)} \\
&= (\rho_i - \sigma_i)^2 + 2\sigma_i(\rho_i - \sigma_i) \\
&= \rho_i^2 - \sigma_i^2.
\end{aligned}$$

Hence, $q_i^2(\tau_i') > p_i^2(\tau_i')$, which is in contradiction with definition of τ_i'. Thus, on the interval $[0, \tau_i]$, $z_i(t) \neq 0$. In particular, $z_i(\tau_i) \neq 0$. Starting from the time τ_i the evader uses the strategy (6)

$$v_i(t) = \begin{cases} 0, & \tau_i \leq t \leq \tau_i + \varepsilon_i, \\ u_i(t - \varepsilon_i), & t > \tau_i + \varepsilon_i, \end{cases} \quad i = 1, 2, ...,$$

where ε_i is a positive number that will be chosen below. Observe that

$$z_i(t) = e^{-\lambda_i t}\left(z_{i0} + \int_0^t e^{-\lambda_i s}(u_i(s) - v_i(s))ds\right) = 0$$

if and only if

$$y_i(t) = z_{i0} + \int_0^t e^{-\lambda_i s}(u_i(s) - v_i(s))ds = 0.$$

Therefore it suffices to show the inequality $y_i(t) \neq 0$, $t \in [\tau_i, \infty)$.

Let $\tau_i \leq t \leq \tau_i + \varepsilon_i$. Then

$$\begin{aligned} y_i(t) &= z_{i0} + \int_0^t e^{-\lambda_i s}(u_i(s) - v_i(s))ds \\ &= y_i(\tau_i) + \int_{\tau_i}^t e^{-\lambda_i s}(u_i(s) - v_i(s))ds \\ &= y_i(\tau_i) + \int_{\tau_i}^t e^{-\lambda_i s}u_i(s)ds. \end{aligned}$$

We have

$$\int_{\tau_i}^t e^{-\lambda_i s}|u_i(s)|ds \leq \varphi_i(\tau_i, t)\left(\int_{\tau_i}^t |u_i(s)|^2 ds\right)^{1/2} \leq \rho_i\varphi_i(\tau_i, \tau_i + \varepsilon_i).$$

If $b - a = \varepsilon_i$, $a \in [0, T]$, then

$$\begin{aligned} \varphi_i(a, b) &= \sqrt{\int_a^b e^{2\lambda_i s}ds} = \sqrt{\frac{1}{2\lambda_i}\left(e^{2\lambda_i b} - e^{2\lambda_i a}\right)} \\ &= e^{\lambda_i a}\sqrt{\frac{1}{2\lambda_i}\left(e^{2\lambda_i(b-a)} - 1\right)} \leq e^{\lambda_i T}\sqrt{\frac{1}{2\lambda_i}\left(e^{2\lambda_i\varepsilon_i} - 1\right)}. \end{aligned}$$

Let the number ε_i be chosen such that

$$\rho_i e^{\lambda_i T}\sqrt{\frac{1}{2\lambda_i}\left(e^{2\lambda_i\varepsilon_i} - 1\right)} \leq |y_i(\tau_i)|/2. \tag{10}$$

Then

$$\rho_i\varphi_i(\tau_i,\, \tau_i + \varepsilon_i) \leq |y_i(\tau_i)|/2,$$

and so

$$|y_i(t)| \geq |y_i(\tau_i)| - \int_{\tau_i}^t e^{-\lambda_i s}|u_i(s)|ds \geq |y_i(\tau_i)|/2, \tau_i \leq t \leq \tau_i + \varepsilon_i.$$

Let now $t > \tau_i + \varepsilon_i$. Then

$$\begin{aligned} y_i(t) &= y_i(\tau_i) + \int_{\tau_i}^{\tau_i+\varepsilon_i} e^{-\lambda_i s} u_i(s)ds + \int_{\tau_i+\varepsilon_i}^{t} e^{-\lambda_i s}(u_i(s) - v_i(s))ds \\ &= y_i(\tau_i) + \int_{\tau_i}^{t} e^{-\lambda_i s} u_i(s)ds - \int_{\tau_i+\varepsilon_i}^{t} e^{-\lambda_i s} u_i(s-\varepsilon_i)ds. \end{aligned} \tag{11}$$

Since

$$\int_{\tau_i+\varepsilon_i}^{t} e^{-\lambda_i s} u_i(s-\varepsilon)ds = \int_{\tau_i}^{t-\varepsilon_i} e^{-\lambda_i s} u_i(s)ds,$$

therefore (11) implies that

$$\begin{aligned} y_i(t) &= y_i(\tau_i) + \int_{\tau_i}^{t} e^{-\lambda_i s} u_i(s)ds - \int_{\tau_i}^{t-\varepsilon_i} e^{-\lambda_i s} u_i(s)ds \\ &= y_i(\tau_i) + \int_{t-\varepsilon_i}^{t} e^{-\lambda_i s} u_i(s)ds. \end{aligned}$$

Hence, it follows from (10) that

$$\begin{aligned} |y_i(t)| &\geq |y_i(\tau_i)| - \int_{t-\varepsilon_i}^{t} e^{-\lambda_i s}|u_i(s)|ds| \\ &\geq |y_i(\tau_i)| - \rho_i \varphi_i(t-\varepsilon_i, t) \geq \; |y_i(\tau_i)|/2, \end{aligned}$$

for all $t \geq \tau_i + \varepsilon_i$. Thus, $|y_i(t)| \geq |y_i(\tau_i)|/2 > 0$ for all $t \geq \tau_i$. In other words, evasion is possible on the interval $[0, T_i)$.

Show that evasion is possible on the interval $[0, T')$, $T' = \sup_{i=1,2,\ldots} T_i$. Let $t \in [0, T')$ be any time. Then by definition of supremum $t \in [0, T_j)$ at some $j \in N$. As proved above $y_j(t) \neq 0$ for $t \in [0, T_j)$, and hence $z_j(t) \neq 0$. In its turn this inequality implies that $z(t) \neq 0$ meaning that evasion is possible on the interval $[0, T')$. Thus, T' defined by (4) is optimal pursuit time. Proof is complete.

4. Conclusion

We have studied a differential game of one pursuer and one evader described by infinite system of first order differential equations. Control functions of pursuer and evader are subject to coordinate-wise integral constraints. Earlier we showed that the pursuit time defined by formula (4) is guaranteed pursuit time. In the present paper, we proved that this time is optimal pursuit time. Moreover, we have constructed optimal strategies of players. The differential game can be extended by studying differential games 1) described higher order differential equations, or 2) with many pursuers.

Acknowledgements
This research was partially supported by the National Fundamental Research Grant Scheme (FRGS) of Malaysia, No. 01-01-13-1228FR.

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